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Two-step method for radiative transfer calculations in a developing pool fire at the initial stage of its suppression by a water spray



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Leonid A. Dombrovsky^{a,b,c,*}, Siaka Dembele^a, Jennifer X. Wen^d, Ivan Sikic^d

^a Department of Mechanical & Automotive Engineering, Kingston University, London SW15 3DW, UK
 ^b Joint Institute for High Temperatures, 17A Krasnokazarmennaya St, Moscow 111116, Russia
 ^c The University of Tyumen, 6 Volodarsky St, Tyumen 625003, Russia
 ^d School of Engineering, University of Warwick, Coventry CV4 7AL, UK

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ABSTRACT

A procedure based on two-step method is suggested to simplify time-consuming spectral radiative transfer calculations in open flames containing scattering particles. At the first step of the problem solution, the P_1 approximation is used to calculate the divergence of radiative flux, and it is sufficient to determine the flame parameters. The second step of solution is necessary to obtain the radiation field outside the flame, and this can be made independently using the ray-tracing procedure and the transport source function determined at the first step. Such a splitting of the complete problem results in much simpler algorithm than those used traditionally. It has been proved in previous papers that the combined twostep method is sufficiently accurate in diverse engineering applications. At the same time, the computational time decreases in about two orders of magnitude as compared with direct methods. An axisymmetric pool fire at the initial stage of fire suppression by a water spray is considered as the case problem. It is shown that evaporating small water droplets characterised by a strong scattering of infrared radiation are mainly located in regions near the upper front of the flame and one can observe the scattered radiation. This effect can be used in probe experiments for partial validation of transient Computational Fluid Dynamics (CFD) simulations.

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1. Introduction

Radiative transfer calculations are the most time-consuming in CFD simulation of fires. Therefore, the use of a simple but sufficiently accurate method for the radiative transfer is critically important to improve the computational costs of CFD codes. A numerical analysis of suppression of fires by water sprays is possible only with the use of spectral models because the optical properties of water droplets cannot be considered on the basis of a gray model [1]. In most cases, there is no need for a very detailed radiation field in CFD calculations. Only the total or integral (over the spectrum) radiative flux divergence is important because this term is needed in the energy equation that couples thermal radiation with other modes of heat transfer. It means that one can consider a differential approach instead of the radiative transfer equation (RTE). Such a choice is really reasonable in the case of combined heat transfer problems including those for fires and combustion

 \ast Corresponding author at: Joint Institute for High Temperatures, 17A Krasnokazarmennaya St, Moscow 111116, Russia.

E-mail address: ldombr@yandex.ru (L.A. Dombrovsky).

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systems [1,2]. The P_1 (the first-order approximation of the spherical harmonics method) [1–4] is the simplest method of this type, and there is a very positive long-time experience of using this approach in diverse multi-dimensional problems [1,5–17]. It appears to be sufficiently accurate in the case when one needs only the divergence of radiative flux in the energy equation. Moreover, it was shown in recent paper [18] that the P_1 error is sometimes less than that of the finite-volume method.

It was demonstrated in early papers [19-21] that P_1 approximation may give incorrect values of radiative flux near the boundaries of the computational region in the case of a strong decrease of temperature or extinction coefficient of the medium towards the region boundaries. Unfortunately, this is the case for open flames. Therefore, the second step of the computational radiative transfer model is necessary. This can be done using the transport approximation [12] and a ray-tracing procedure for the RTE with the transport source function determined at the first step of solution. In particular, this combined two-step method presented in papers [5,19,20] was successfully employed in recent paper [17] to calculate radiative heat transfer from supersonic flow with suspended particles to a blunt body.

Nomenclature

Α	area of boundary surface	3	spectral emissivity
а	radius of droplet	ζ	coefficients in Eq. (25c)
С	specific heat capacity	η	dynamic viscosity
D	radiation diffusion coefficient	k	index of absorption
Ε	normalized coefficient	λ	wavelength of radiation
f_{v}	volume fraction of droplets	$\bar{\mu}$	asymmetry factor of scattering
F	function introduced by Eq. (10)	μ_0	cosine of scattering angle
G	irradiation	ξ	coefficient in Eq. (25a)
g	acceleration of gravity	ρ	density
Ĩ	radiation intensity	σ	scattering coefficient
Κ	coefficient introduced by Eq. (24b)	τ	optical thickness
k	thermal conductivity	γ	functional (9)
L	latent heat of evaporation	$\tilde{\Phi}$	scattering phase function
1	coordinate along the ray	φ	coefficient introduced by Eq. (25c)
т	complex index of refraction	Ψ	coefficient introduced by Eq. (24b)
п	index of refraction	$\overrightarrow{\Omega}$	unit vector of direction
Q	efficiency factor		
\overrightarrow{q}	radiative flux	Subscrip	ots and superscripts
r	radial coordinate	a	absorption
\overrightarrow{r}	spatial coordinate	b	blackbody
S	radiative source function	cl	cloud
S	flame thickness along the ray	d	droplet
Т	temperature	ea	equilibrium
t	time	g	292 24
и	velocity	i	spectral band number
x	diffraction parameter	max	maximum
V	volume	rel	relaxation
W	divergence of radiative flux	S	scattering
Ζ	axial coordinate	sat	saturation
		tr	transport
Greek sy	mbols	w	water, wall
α	absorption coefficient	λ	spectral
D			
р	extinction coefficient	0	initial value
Ρ γ	extinction coefficient coefficient introduced by Eq. (24b)	0	initial value

The present study is focused on applicability of the earlier developed combined two-step method to the flames typical of pool fires. The small developing flame just before its suppression by water sprays and also at the initial probe stage of suppression is considered. It should be noted that radiation scattering by evaporating droplets is important even at the beginning of fire suppression and this effect is favourable for the P_1 accuracy because of more smooth angular dependence of the spectral radiation intensity [1].

Following paper [22], we consider infrared radiation outside the absorption band of water to focus on the effect of radiation scattering by water droplets. The simple box model is used for spectral absorption coefficient of molecular gases, but this choice is not critical for the present study because it is focused on a general approach for the radiative transfer calculations and also on infrared scattering by evaporating water droplets outside the absorption bands of water. A numerical solution for the conventional axisymmetric flame (after averaging the numerical results for the 3-D flow field) is considered because this simplification in the flame data is acceptable for the present study.

2. An estimate of the radiation effect on the flow field calculations

The effect of radiation on the flow field of a small developing flame before the flame suppression is considered. The fire scenario for the case problem is based on a 28 cm diameter methane gas flame, burning in the open quiescent environment with a heat release rate of 53 kW. The LES (large eddy simulation) CFD simulation was carried out using an in-house version of FireFOAM code [23], compatible with OpenFOAM version 2.2.x. The combustion sub-model used in this code is based on the Eddy Dissipation Concept (EDC) proposed in paper [24] for RANS (Reynolds-Averaged Navier–Stokes) applications and extended to LES in [25]. The radiative transfer equation is solved using the FVM code, which is open accessible. Gas radiation is treated with the box model approach based on the exponential wide band model to calculate the equivalent band absorption coefficient [26]. The flow field in the present study was computed using FireFoam.

The calculated transient flame is three-dimensional, but deviations of the main parameters from the axisymmetric case are not significant. Therefore, we consider similar axisymmetric fields obtained by an averaging in a selected axial cross section of the real developing flame at t = 1 s. A comparison of two temperature fields shown in Fig. 1 indicates that the effect of thermal radiation on the flame formation is relatively small at early stages of flame development. In particular, the maximum temperature in the flame is underestimated by about 1.5% in the calculation without taking into account radiation. Nevertheless, thermal radiation should be taken into account because of its considerable effect on the parameters at the upper "head" of the flame (compare Fig. 1a and b). In light of this observation, rather than using detailed and time-consuming methods for radiative transfer calculations, a much simpler P_1 approximation seems to be reasonable. Of course, the error of this simple approach should be additionally verified to be sure that it is acceptable for the developing flame



Fig. 1. The gas temperature field in the model flame: a - the calculation without account for thermal radiation, b - the complete calculation.

calculations. As to the radiative flux from the flame, it can be calculated using a ray-tracing procedure at the second step of solution independently of the CFD calculations.

3. Two-step method for radiative transfer calculations

The general theoretical model for transfer of thermal radiation in a flame should be based on the radiative transfer equation (RTE) which takes into account emission, absorption, and scattering of radiation in the medium. A special feature of the fire suppression problem is the anisotropic scattering of radiation by relatively cold water droplets. In this case, the RTE can be written as follows [1] (hereafter, we omit the subscript " λ " in the medium radiative properties for brevity):

$$\vec{\Omega}\nabla I_{\lambda}(\vec{\mathbf{r}},\vec{\Omega}) + \beta I_{\lambda}(\vec{\mathbf{r}},\vec{\Omega}) = \frac{\sigma}{4\pi} \int_{(4\pi)} I_{\lambda}(\vec{\mathbf{r}},\vec{\Omega}') \Phi(\vec{\Omega}'\cdot\vec{\Omega}) d\vec{\Omega}' + \alpha_{g} I_{b}[T_{g}(\vec{\mathbf{r}})]$$
(1)

The physical meaning of Eq. (1) is evident: variation of the spectral radiation intensity, I_{λ} , in direction $\vec{\Omega}$ takes place due to selfemission of thermal radiation by gases (the last term, $I_{\rm b}(T)$ is the Planck function), extinction by absorption and also by scattering in other directions (the second term), as well as due to scattering from other directions (the integral term). The spectral extinction coefficient β is a sum of the absorption coefficient, $\alpha = \alpha_{\rm g} + \alpha_{\rm d}$, (both gas and water droplets contribute to absorption) and the scattering coefficient, σ , of a conventional continuous medium of droplets. These spectral coefficients and the scattering phase function, Φ , depend on spatial coordinate \vec{r} .

The integration of RTE (1) over all values of the solid angle gives the following important expression for divergence of the spectral radiative flux [1]:

$$\nabla \vec{q}_{\lambda} = 4\pi \alpha_{\rm g} I_{\rm b}(T_{\rm g}) - \alpha G_{\lambda} \tag{2}$$

which can be treated as a local balance of the spectral radiation energy. The spectral radiative flux and irradiation function are defined as follows:

$$\vec{\mathbf{q}}_{\lambda}(\vec{\Omega}) = \int_{(4\pi)} I_{\lambda}(\vec{\mathbf{r}},\vec{\Omega}) \vec{\Omega} d\vec{\Omega} \quad G_{\lambda}(\vec{\mathbf{r}}) = \int_{(4\pi)} I_{\lambda}(\vec{\mathbf{r}},\vec{\Omega}) d\vec{\Omega}$$
(3)

The first term in the right-hand side of Eq. (2) is the emission, whereas the second term is the radiation absorption by both the gases and water droplets.

A complete and accurate solution to the RTE in scattering media is a very complicated task. One can find many studies in the literature on specific numerical methods developed to obtain more accurate spatial and angular characteristics of the radiation intensity field. Several modifications of the discrete ordinates method (DOM), finite-volume method (FVM), and statistical Monte Carlo (MC) methods are the most popular tools employed by many authors (see [1–4] for the details). The main difficulty of accurate radiative transfer calculations is related with the angular dependence of radiation intensity. Fortunately, this angular dependence appears to be rather simple in many important applied problems. It enables one to use this property of solution to derive relatively simple but fairly accurate differential approximations.

All the differential approximations for RTE are based on simple assumptions concerning the angular dependence of the spectral radiation intensity I_{λ} . These assumptions enable us to deal with a limited number of functions $I_{\lambda}^{i}(\vec{r})$ instead of function $I_{\lambda}(\vec{r}, \vec{\Omega})$ and turn to the system of the ordinary differential equations by the use of integration of RTE. With the use of the spherical harmonics method, the spectral radiation intensity is presented in a series of spherical functions. In the first approximation of this method, P_{1} , the following linear dependence is assumed [1]:

$$I_{\lambda}(\vec{r},\vec{\Omega}) = \frac{1}{4\pi} \left[G_{\lambda}(\vec{r}) + 3\vec{\Omega}\vec{q}_{\lambda}(\vec{r}) \right]$$
(4)

By multiplying Eq. (1) by $\vec{\Omega}$ and integrating it over a solid angle, one can find that the radiative flux is related to the irradiation by equation

$$\vec{\mathbf{q}}_{\lambda} = -D\nabla G_{\lambda} \tag{5}$$

where $D = \frac{1}{3\rho_{tr}}$ is the spectral radiation diffusion coefficient, $\beta_{tr} = \alpha + \sigma_{tr}$ is the transport extinction coefficient, $\sigma_{tr} = \sigma.(1 - \bar{\mu})$ is the transport scattering coefficient, $\bar{\mu}$ is the asymmetry factor of scattering introduced as follows:

$$\bar{\mu} = \frac{1}{4\pi} \int_{(4\pi)} I_{\lambda} (\vec{\Omega}' \cdot \vec{\Omega}) \Phi(\vec{\Omega}' \cdot \vec{\Omega}) d\vec{\Omega}'$$
(6)

Note that Eq. (5) is obtained for an arbitrary scattering function but it is the same as that for the transport approximation when this function is replaced by a sum of the isotropic component and the term describing the peak of forward scattering [1,12]:

$$\Phi(\mu_0) = (1 - \bar{\mu}) + 2\bar{\mu}\delta(1 - \mu_0) \tag{7}$$

where $\mu_0 = \vec{\Omega} \cdot \vec{\Omega}'$ is the cosine of the angle of scattering. It means that P_1 approximation is insensitive to details of the scattering function and the asymmetry factor of scattering is the only parameter of scattering anisotropy taken into account in this approach. The latter does not lead to considerable errors in radiative heat transfer calculations in the practically important case of multiple scattering [1,12].

Substituting Eq. (5) into the radiation energy balance (2) we obtain the nonhomogeneous modified Helmholtz equation for the spectral irradiation:

$$-\nabla (D\nabla G_{\lambda}) + \alpha G_{\lambda} = 4\pi \alpha_{\rm g} I_{\rm b}(T_{\rm g}) \tag{8}$$

The Marshak boundary conditions are used to complete the P_1 boundary-value problem [1]. One can also proceed to the variational formulation of the problem when the solution yields the minimum of the following functional [1,5]:

$$\chi_{\lambda} = \int_{V} \left[\frac{D}{2} \nabla G_{\lambda} + \frac{\alpha}{2} G_{\lambda}^{2} - 4\pi \alpha_{g} I_{b}(T_{g}) G_{\lambda} \right] dV + \int_{A} \frac{\varepsilon_{w}}{2(2 - \varepsilon_{w})} \left[\frac{G_{\lambda}^{2}}{2} - 4\pi I_{b}(T_{w}) G_{\lambda} \right] dA$$
(9)

where T_w and ε_w are the temperature and hemispherical emissivity of boundary surfaces. The first integral in Eq. (9) corresponds to the differential Eq. (8), which is true in the computational region, whereas the second integral takes into account the conditions at the region boundary. The order of the differential operator in Eq. (9) is reduced by one compared with Eq. (8). As a result, the local linear approximations of function G_{λ} in triangle finite elements are sufficient. The final-element method (FEM) [27,28] can be used to solve the variational problem for functional (8) as it was done in papers [9,17,29]. The FEM is very convenient because this method is flexible and applicable for computational regions of complex shape.

It should be recalled that the box spectral model with constant values of absorption coefficient in a set of spectral bands is used in the current version of the CFD code FireFOAM. According to this spectral model, the Planck function, $I_{\rm b}(T)$, in Eq. (8) is replaced by the following integral:

$$F_{b,j}(T) = \int_{\lambda_{1,j}}^{\lambda_{2,j}} I_b(T) d\lambda \tag{10}$$

As a result, the integral value of irradiation, G_j , for every band of number "j" is obtained instead of the spectral irradiation. A contribution of the spectral band to the divergence of the integral (over the spectrum) radiative flux is given by the following relation:

$$\nabla \vec{\mathbf{q}}_{j} = 4\pi \alpha_{g,j} F_{b,j}(T_g) - \alpha_j G_j \tag{11}$$

Let consider the computational data for CO₂ absorption band with wavelength boundaries of $\lambda_1 = 4.1494 \,\mu\text{m}$ and $\lambda_2 = 4.5851 \,\mu\text{m}$ for the model flame with temperature field shown in Fig. 1b. The absorption coefficient in this spectral band calculated with the box gas radiation model is plotted in Fig. 2.

The radial optical thickness, τ , of the hottest region of the flame without water droplets is easily estimated by multiplying the absorption coefficient, $\alpha_{g,j}$, by the geometrical thickness, Δr , of this thin region. The calculations carried out by the authors show that a value of τ that is much smaller than unity is obtained. It means that the upper estimate of the radiative heat losses based on the optically thin approximation may be applicable in this scenario specific to the initial development stage of the fire.

The non-uniform triangulation of the computational region, which includes the main part of the temperature field of Fig. 1b, is shown in Fig. 3. The left boundary of the region is the surface of the liquid fuel. The smaller areas of finite elements were made in the region of more optically dense medium. The total number of triangular elements is $2 \times 40 \times 40 = 3200$. As usually, the radial coordinates of all the nodes were increased by the first radial interval of the mesh to avoid the formal difficulties in numerical calculations near the axis [1]. The finite elements near the front of the developing flame should be relatively small (as it is shown in Fig. 3) because of an expected increase in extinction in this part of the flame containing numerous absorbing and scattering water droplets. Note that triangular elements at a small distance from the liquid fuel look not good because of a significant difference between the radial and axial dimensions of the triangles. However,



Fig. 2. Absorption coefficient calculated for the CO₂ absorption band.



additional calculations showed that this weakness is not critical in the case of the optically thin developing flame.

The following dimensionless function is calculated using various approaches:

$$\bar{W}_{j} = \frac{\nabla \vec{q}_{j}}{4\pi\alpha_{g,j}(T_{g,max})F_{b,j}(T_{g,max})}$$
(12)

This function is convenient to compare the results of calculations with those obtained in the optically thin limit when there is no need to solve the equations presented above because the following simplest relation is true for the radiative flux divergence [3,4]:

$$\nabla \vec{q}_{j} = 4\pi \alpha_{gj} F_{bj}(T_g) \tag{13}$$

The results of calculations are presented in Fig. 4. The FVM calculations are carried out using FireFoam. One can see that the difference between the computational data obtained using the optically thin approximation, P_1 method, and FVM are similar to each other and the first of these approaches gives an upper estimate of radiative heat losses. The calculations based on P_1 and FVM give very close values of the radiative flux divergence, whereas the computational time of the P_1 calculations is about two orders of magnitude less than that of the regular FVM procedure used in the CFD code Fire-Foam. The results obtained confirm good accuracy of P_1 approximation which can be employed for very fast and reliable radiation calculations for the developing flames and can be recommended for implementation into any general CFD code. It should be recalled that P_1 is even more accurate in the case of a scattering medium [1] and can be used in radiative heat transfer calculations for the flames with water droplets as applied to the flame suppression problem.

The P_1 model is considered for calculations of the radiation term in the energy equation only. This is really important for the CFD flame calculations, but insufficient to determine the radiative flux outside the flame. Fortunately, the transport approximation can be employed to simplify the RTE, which can be written in the same way as that for isotropic scattering, i.e., with $\Phi = 1$ [1,5,12]:

$$\vec{\Omega} \nabla I_{\lambda}(\vec{r}, \vec{\Omega}) + \beta_{tr} I_{\lambda}(\vec{r}, \vec{\Omega}) = S_{\lambda}(\vec{r}) \quad S_{\lambda}(\vec{r}) = \frac{\sigma_{tr}}{4\pi} G_{\lambda}(\vec{r}) + \alpha_{g} I_{b}(T_{g})$$
(14)

where S_{λ} is the so-called source function. The spectral radiation intensity can be calculated by direct integration of Eq. (14). This procedure is very simple in the case of an open flame when no radiation illuminates the flame surface and the formal solution along the ray \vec{s} is as follows:



Fig. 4. The divergence of radiative flux in the CO₂ absorption band: a – optically thin approximation, b – P₁ calculation, c – FVM calculation.

$$I_{\lambda}(s) = \int_{o}^{s} S_{\lambda}(l) exp\left[\int_{o}^{l} \beta_{tr}(l') dl' - \int_{o}^{s} \beta_{tr}(l') dl'\right] dl$$
(15)

Eq. (14) can be rewritten as applied to a set of spectral bands considered in the box model. In the combined two-step method of the present paper, the source function is calculated at the first step using of the P_1 solution for irradiation G_{λ} . After that, a ray-tracing procedure for the transport RTE solution should be used at the second step to calculate accurately the radiative flux to an arbitrary surface outside the flame, if necessary. It is interesting that there is a potential possibility of comparison of computational predictions with experimental data for the radiation of pool fire flames [30].

4. Computational model for the probe stage of fire suppression

The effects of water sprays on the flow field parameters of the flame should be taken into account in computational studies for the regular developed regime of fire suppression, and this is an important engineering problem [31–34]. However, the problem is significantly simplified at the probe stage of fire suppression suggested in [22] because this preliminary stage of fire suppression is characterized by a very small flow rate of water. The observations at the probe stage could show us some details of behavior of evaporating water droplets in the flame.

A short-time probe stage of fire suppression is considered in the present paper. Therefore, the motion, heating, and evaporation of single droplets can be studied without account for any feedback effects. The radiative power absorbed by droplets is neglected because it is much less than the heat supplied to the droplets by convective heat transfer from hot combustion gases in the flame. In more detailed calculations, the effect of radiation can be taken into account as it was done in recent papers on shielding of fire radiation by water mists [35,36] and also in the problem of a space vehicle shielding from solar radiation by a cloud of sublimating particles [37].

The interaction of water sprays with fires has been modeled computationally in many papers during the last two decades [38–45], but there is no need to discuss here the state-of-the-art in this field because of the main focus on the radiative transfer problem. Of course, there is a size distribution of water droplets at the initial cross section of the spray. However, the assumption of independent behavior of droplets makes it possible to consider a set of solutions for droplets of different size in the expected range of the initial droplet size distribution. The trajectories of droplets moving along the flame axis are considered. The radial component of gas velocity in a flame is much less than the axial gas velocity. In addition, the initial direction of the droplet motion is also close to the axial one. This makes the 1-D approach for the dynamic and thermal interaction of water droplets and flame to be acceptable for the computational estimates of the main effects. To simplify the problem, we do not consider also the secondary effects of the vapor coming from evaporating droplets on both the drag force and convective heat flux to the droplet. This is the usual assumption used in computational models of this kind. The initial problem for the droplet motion is as follows [18,41,46]:

$$\frac{dz_d}{dt} = u_d \qquad z_d(0) = z_0 \tag{16a}$$

$$\frac{du_{\rm d}}{dt} = \frac{3C_{\rm D}}{8a} \frac{\rho}{\rho_{\rm w}} (u_{\rm g} - u_{\rm d}) |u_{\rm g} - u_{\rm d}| - g \qquad u_{\rm d}(0) = -u_{\rm d0}$$
(16b)

$$C_{\rm D} = \frac{24(1+0.15 {\rm Re}_{\rm d}^{0.687})}{{\rm Re}_{\rm d}} \qquad {\rm Re}_{\rm d} = 2\rho |u - u_{\rm d}| \frac{a}{\eta} \tag{16c}$$

where the subscript "w" refers to water, u is the axial component of velocity, a is the droplet radius, C_D is the drag coefficient, Re is the Reynolds number. It is assumed that water droplets are first heated up to the saturation temperature:

$$\frac{dT_{\rm d}}{dt} = \frac{1.5 \,{\rm Nu}\,k}{\rho_{\rm w} c_{\rm w} a^2} (T_{\rm g} - T_{\rm d}) \ T_{\rm d}(0) = T_0 \qquad t < t_{\rm sat}$$
(17a)

$$Nu = 2 + 0.6Re_d^{\frac{1}{2}}Pr^{\frac{1}{3}} \qquad Pr = \frac{\eta c}{k}$$
(17b)

(Nu and Pr are the Nusselt and Prandtl numbers) and then evaporated according to the following equation:

$$\frac{da}{dt} = -\mathrm{Nu}\frac{k(T_{g} - T_{w,sat})}{2a\rho_{w}L_{w}} \qquad a(t_{sat}) = a_{0} \quad t \ge t_{sat}$$
(18)

where L_w is the latent heat of evaporation of water. A similar model has been also used in papers [22,35,36]. This simplified evaporation model is also sufficient for qualitative estimates of the present paper because of very fast heating of moving droplets and large molar fraction of water vapour in the flame. It is not necessary at the moment to consider sophisticated evaporation models like that developed in paper [47]. A transfer from the droplet heating to its evaporation is given by the following simple equation:

$$T_{\rm d}(t_{\rm sat}) = T_{\rm w,sat} \tag{19}$$

Obviously, the local relative volume fraction of water droplets can be calculated as:

$$\bar{f}_{\nu} = \frac{f_{\nu}}{f_{\nu 0}} = \left(\frac{u_{\rm d0}}{u_{\rm d}}\right) \left(\frac{a}{a_0}\right)^3 \tag{20}$$

Let consider the axial profiles of a gas temperature, $T_{\rm g}$, and velocity, u_{σ} , on the axis of symmetry (r = 0) and also along the lines of r = 0.05 m, 0.1 m, and 0.15 m for the flame shown in Fig. 1b. The profiles plotted in Fig. 5 in a combination with temperature field presented in Fig. 1b makes clear a strong non-uniformity of the developing flame. A very hot narrow circular region in a vicinity of a ring with radius of r = 0.05 m and height between z = 0.31 m and 0.46 m (red curves in Fig. 5a and b) moves upward with a relatively high speed of about 4 m/s. At the same time, there is a relatively thick region of a cold and slowly moving gas near the flame axis at z > 0.5 m and r < 0.08 m. Moreover, a downward motion of a gas is observed in a part of this region. One can see that both the gas temperature and velocity profiles at r = 0.1 m and 0.15 m are quite different from those at r = 0 and 0.05 m. The above described specific structure of the flame is a motivation of subsequent attention to behaviour of water droplets moving in the flame along different axial lines.

Before proceeding to the computational modeling of motion of water droplets in a gas flow, it is interesting to estimate the so-called relaxation time, t_{rel} , of the droplets. This can be done on the basis of the Stokes law which is true in the case of $\text{Re}_{d} < 1$ [48]:

$$t_{\rm rel} = \frac{2}{9} \frac{\rho a^2}{\eta} \tag{21}$$

Eq. (21) gives very small values of $t_{\rm rel} < 0.25$ ms for water droplets of radius $a < 100 \ \mu$ m. It means that water droplets should be supplied at a small axial distance from the flame and the initial velocity of droplets should be sufficient to reach a hot region inside the flame.

Obviously, the effect of initial conditions decreases along the droplet trajectory, and the equilibrium position of a droplet is characterized by the same absolute values of two opposite forces: the force of gravity and the drag force which supports the droplet. In the case of a relatively small evaporating droplet, this condition

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Fig. 5. The axial profiles of (a) temperature and (b) axial component of velocity. of gaseous combustion products in the model flame.

can be written using the Stokes law. The resulting value of the gas axial velocity is as follows:

$$u_{\rm g} = \frac{2}{9} \frac{\rho_{\rm w} {\rm g} a^2}{\eta} \tag{22}$$

Having substituted $\frac{\eta}{\rho_w} = 1.8 \times 10^{-8} \frac{s}{m^2}$ and $a = 40 \,\mu\text{m}$ to Eq. (22), we obtain $u_g \approx 0.2 \,\text{m/s}$. One can see in Fig. 5b that this value of the axial velocity is observed near the flame axis at $z_{eq} = 0.52 \,\text{m}$. It is interesting to compare this estimate with detailed numerical calculations.

The numerical results for the current radius of evaporating water droplets along their trajectories in the case of the droplet initial position of $z_0 = 0.75$ m and various distances, r_0 , from the flame axis are presented at the in Fig. 6. The same effect is observed at different values of r_0 : one can choose the value of u_0 and the range of a_0 to observe focusing of all the droplets in small local regions where the equilibrium condition (22) is satisfied. It should be noted that the above estimate of z_{eq} for the droplets moving along the flame axis ($r_0 = 0$) is in good agreement with the calculations shown in Fig. 6a. The calculations for droplets moving along the flame axis don't show their complete evaporation because of a cold gas area near the axis. This is a special case when the thermal radiation of a hot flame should be taken into account in calculations of the droplet evaporation. On the contrary, large water droplets supplied at $r_0 = 0.1$ m and $r_0 = 0.15$ m can penetrate into the hot region and these droplets are totally evaporated during their backward motion at a small distance from this hot region.

It is physically clear that the volume fraction of water droplets increases dramatically in the local regions of their collection and



Fig. 6. Variation of radius of droplets along their trajectories at three initial positions of the droplets: (a) $r_0 = 0$ ($u_{d0} = 20$ m/s), (b) $r_0 = 0.1$ m ($u_{d0} = 10$ m/s), and (c) $r_0 = 0.15$ m ($u_{d0} = 15$ m/s).

total evaporation. Strictly speaking, the assumption of independent motion of single droplets may be incorrect in the vicinity of these regions. However, following [20], this effect is not considered in the present paper.

5. Optical properties of water droplets

The normalized coefficients of absorption and transport scattering (per unit volume fraction of water) for independently absorbing and scattering droplets with radius *a* can be determined as follows:

$$E_{\rm a} = 0.75 \frac{Q_{\rm a}}{a} \qquad E_{\rm s}^{\rm tr} = 0.75 \frac{Q_{\rm s}^{\rm tr}}{a}$$
(23)

According to Mie theory [49–51], the efficiency factors of absorption, Q_a , and transport scattering, Q_s^{tr} depend on both the complex index of refraction of water, m = n - ik, and diffraction parameter $x = \frac{2\pi a}{\lambda}$. The following approximate relations suggested in [52] are used in the calculations instead of the Mie theory:

$$Q_{a} = \frac{4n}{(n+1)^{2}} [1 - \exp(-4kx)] \qquad Q_{s}^{tr} = K. \begin{cases} \frac{\psi}{5} \text{ when } \psi \leqslant 5\\ \left(\frac{5}{\psi}\right)^{\gamma} \text{ when } \psi > 5 \end{cases}$$
(24a)

 $K = 1.5n(n-1)\exp(-15k) \ \gamma = 1.4 - \exp(-80k) \qquad \psi = 2x(n-1)$ (24b)

The spectral optical constants of water, n and k, can be found in early papers [53,54]. According to [55,56], we neglect a weak temperature dependence of the refractive index. It means that the most important value of transport scattering coefficient in the range of water semi-transparency can be calculated using the values of optical constants of water at room temperature. The calculated values of E_a and E_s^{tr} presented in Fig. 7 should be multiplied by the volume fraction of droplets to obtain the local values of the spectral absorption and transport scattering coefficients: $\alpha_d = f_v E_a$ and $\sigma_{tr} = f_v E_s^{tr}$. One can see that radiation absorption by water droplets is significant in the main near-infrared absorption band of water. It contributes to water evaporation and flame suppression. The scattering by small droplets is large over the spectrum excluding the absorption band, where the anomalous dispersion takes place [57,58]. The hypothesis of independent scattering does not work in the region of strong evaporation where the distance between the neighboring droplets may be comparable with the wavelength. The dependent scattering may lead to significant changes in optical properties of the medium [59–62]. However, this effect is a complex problem of physical optics and it is beyond the scope of the present paper. Therefore, the ordinary relations of independent scattering by single droplets are used in the estimates.

It should be noted that there are two serious assumptions of the suggested theoretical model and both of them are related with an expected high local volume fraction of small water droplets. Strictly speaking, one cannot consider independent motion of these

droplets and also independent scattering of infrared radiation by water droplets in some local regions of the flame. The resulting overall error of the simplified computational model depends strongly on water flow rate at the probe stage of the flame suppression. One can decrease this error in the experiments using additional probe-stage regimes with much lower flow rate of water.

6. Effect of radiation scattering by evaporating water droplets

The computational data presented in Fig. 7 confirm that the above choice of the representative CO₂ absorption band at wavelength about 4.3 µm will enable us to estimate the effect of scattering of the flame radiation by evaporating water droplets at the probe stage of fire suppression. Note that contributions of soot to both absorption and scattering of radiation are negligible in this spectral range [63–66]. The absorption of thermal radiation by droplets is very small in this range whereas the normalize transport scattering coefficient is very high: $E_s^{tr} = 300 \text{ cm}^{-1}$ at a = 10 µm and $E_s^{tr} \approx 1000 \text{ cm}^{-1}$ at a = 5 µm (see Fig. 7b). Following [67], the emissivity of the liquid fuel surface $\varepsilon_w = 0.6$ was used in the calculations. The latter does not mean that thermal radiation of relatively cold liquid fuel is important. However, we take into account a partial reflection of the flame radiation from the surface of liquid fuel.

It was shown in the previous section of the paper that relatively small evaporating droplets of water are collected mainly at a small distance from the front of a developing flame. According to Eq. (20) the volume fraction, f_v , of slowly moving droplets in this thin region is very high and the absolute value of is directly proportional to the flow rate of water in water spray. The maximum value of $f_{v,max} = 10^{-3}$ and constant value of $E_s^{tr} = 800 \text{ cm}^{-1}$ are assumed for the cloud of droplets in subsequent calculations. To estimate the effect of infrared scattering by water droplets, the following analytical relations were used for the spatial variation of volume fraction of droplets near the flame with temperature field shown in Fig. 1b:

$$f_{v} = f_{v,\max}\left[1 - \frac{\left(\frac{r}{r_{\max}} - 1\right)^{2}}{2}\right] exp\left[\xi \cdot \left(1 - \frac{z}{z_{cl}}\right)\right] \qquad z_{cl} = z_{\min}\varphi(r)$$
(25a)



Fig. 7. Normalized (a) absorption and (b) transport scattering coefficients of water droplets.



Fig. 8. The normalized radiative source function: a – without water droplets, b – with water droplets.

$$r_{\rm max} = 0.1 \ {\rm m} \qquad \xi = 10 \ {\rm m}^{-1} \qquad z_{\rm min} = 0.5 \ {\rm m} \qquad (25b)$$

$$\varphi(r) = \begin{cases} 1, & r < 0.05 \text{ m} \\ 1 + \zeta_1 r, & 0.05 \leqslant r < 0.1 \text{ m} \\ 1.4 - \zeta_2 r, & 0.1 \leqslant r < 0.2 \text{ m} \end{cases} \quad \zeta_1 = 3 \text{ m}^{-1} \quad \zeta_2 = 1 \text{ m}^{-1}$$
(25c)

It is assumed that scattering by water droplets in the region of $z < z_{cl}$ and also at r > 0.2 m is negligibly small.

As discussed above, the observed radiation from the flame is determined by the radiative source function. Therefore, the effect of infrared scattering by water droplets on the source function is considered. Following the above normalization, the source function for the transport RTE calculated using the P_1 solution for irradiation is divided by the maximum value of the source function calculated using the optically thin approximation. The finiteelement triangulation of the computational region shown in Fig. 5 was used in the calculations. The resulting field of dimensionless value of \overline{S} calculated using P_1 approximation is presented in Fig. 8. One can see that a cloud of small evaporating droplets leads to significant changes in the field of radiative source function just in front of the developing flame. This effect is expected to be interesting for the infrared diagnostics of transient flames at the probe stage of fire suppression. The latter can be used for partial validation of CFD simulations. To the best of our knowledge, there is no appropriate data in the literature on infrared measurements of thermal radiation of a flame suppressed by a jet of water droplets. However, such an experimental work would be interesting.

7. Conclusions

A procedure based on the earlier developed combined two-step method is suggested to simplify time-consuming spectral radiative transfer calculations in transient pool fires during their suppression using water sprays. A comparison with the reference numerical solution indicates that the accuracy of the P_1 approximation used at the first step of the method is sufficient for the calculations of the radiative flux divergence. This will lead to a significant simplification of the radiative module in the CFD code and much faster radiation calculations. This result can be considered as an important stage in improving the fire radiation modeling.

The calculations of motion, heating, and evaporation of water droplets at the probe stage of fire suppression indicated the local areas of focusing of moving droplets before their total evaporation in the flame. The effect of infrared radiation scattering by these small droplets on the source function responsible for the radiation from the flame is considerable. It is important that positions of the local highly-scattering areas are very sensitive to the flame parameters. This can be used in probe experiments for partial validation of transient CFD simulations.

Conflict of interests

None declared.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.ijheatmasstransfer. 2018.07.095.

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