A generalized analytical model for radiative transfer in vacuum thermal insulation of space vehicles

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Abstract

The previously developed spectral model for radiative transfer in vacuum thermal insulation of space vehicles is generalized to take into account possible thermal contact between a fibrous spacer and one of the neighboring aluminum foil layers. An approximate analytical solution based on slightly modified two-flux approximation for radiative transfer in a semi-transparent fibrous spacer is derived. It was shown that thermal contact between the spacer and adjacent foil may decrease significantly the quality of thermal insulation because of an increase in radiative flux to/from the opposite aluminum foil. Theoretical predictions are confirmed by comparison with new results of laboratory experiments.

Keywords: Radiative transfer, Transport approximation, Thermal insulation, Fibrous spacer, Vacuum experiment

1. Introduction

The paper is concerned with radiative heat transfer as applied to the Thermal Control Systems (TCS) of spacecrafts. The need in a TCS is dictated by the technological/functional limitations and reliability requirements of all equipment used onboard a spacecraft and, in the case of manned missions, by the need to provide the crew with a suitable living/working environment. Almost all sophisticated equipment has specified temperature ranges in which it will function correctly. The role of the TCS is therefore to maintain the temperature and temperature stability of every item onboard the spacecraft within those pre-defined limits during all mission phases and thereby using a minimum of spacecraft resources.

In previous papers of co-authors the thermal properties of a multilayered thermal-insulating blanket (MLI), which is a screen-vacuum thermal insulation as a part of the TCS for perspective spacecrafts, have been estimated. Heat transfer in the MLI was analyzed on the basis of a theoretical model validated using the laboratory heat flux measurements and the inverse heat transfer problem technique. A transient nature and possible non-linearity of heat transfer can be referred to special features of thermal conditions of modern space structures and especially TCS. These factors considerably reduce the possibility of using some traditional theoretical and experimental methods. Therefore, it is important to develop new approaches to thermal engineering studies and carry out experimental studies under conditions similar to full-scale flight tests.

The particular objective of the present paper is two-fold: (1) to generalize the previously developed radiative transfer model and (2) to compare theoretical predictions with new laboratory experimental data. The methodology of the present paper is based on assumption of an isothermal thin spacer and continuous approach for the radiative transfer in a layered fibrous material of the spacer. The use of slightly modified two-flux approximation enables one to simplify the problem and obtain an analytical solution which is convenient for the engineering estimates. The previously developed vacuum thermal installation is used to measure the heat flux through the vacuum insulation at different conditions and validate the model suggested.

2. Theoretical model

The theoretical model suggested in the present paper is free from the main assumption of paper on the absence of thermal contact between the spacer and the neighboring aluminum foil.
layers. At the same time, the assumptions of a small optical thickness of a fibrous spacer and random orientation of fibers in planes remain without changes. The latter is sufficient to neglect the temperature difference across the spacer. Of course, the maximum estimate of an effect of thermal contact between the foil and spacer can be obtained by ignoring possible local gaps between them. It means that the temperature of the fibrous spacer is assumed equal to the temperature of the adjacent aluminum foil. Strictly speaking, one can take into account a temperature difference across the spacer. This can be done using the approaches developed in papers [3–7]. At the same time, the maximum estimate is considered in the model suggested when this temperature difference is expected to be negligible because of a very small geometrical thickness of ordinary spacers. This is the main simplification which will be evaluated by comparison with the experimental data. Note that the effect of an isothermal spacer is in an increase of spectral emissivity of the foil-spacer system and the solution to the problem under consideration does not depend on which foil (relatively cold or hot) is in the thermal contact with the spacer.

A representative fragment of the vacuum insulation is considered. This fragment consists of two parallel aluminum foils and a flat fibrous spacer between the foils. The spacer is made of several layers of quartz fibers, and the fibers are randomly positioned and oriented in every layer. So, the fibrous medium of a single spacer is the only participating medium between the foils. For brevity, the following designation is used in subsequent analysis:

\[ f_j = n B_\lambda(T_f) \quad j = 1, 2 \quad \text{(1)} \]

where \( j \) is the foil number (\( T_1 > T_2 \)) and \( B_\lambda(T) \) is the Planck function. The spectral radiative flux in a plane-parallel single layer of the vacuum insulation is determined as follows (hereafter the subscript \( \lambda \) for spectral quantities is omitted for brevity):

\[ q = (f_1 - f_2)(1/r_1 + 1/r_2 - 1) \quad \text{(2)} \]

Assume for definiteness that the spacer contacts with the hotter foil (\( T_f = T_p \), subscript "p" refers to the spacer). In this case \( r_2 = n_p \) (subscript "p" refers to the foil), whereas the value of \( r_1 \) taking into account the spacer effect should be obtained.

The principal assumption of the present paper is that a continuous approach is applicable to radiative transfer in a thin semitransparent fibrous spacer. This assumption may lead to quantitative errors in the limit of a spacer containing only few flat layers of fibers, but the obvious advantage of the continuous approach, which enables one to use a simple differential approximation for the radiative transfer, is too attractive to be ignored. In addition, the limiting case of a very thin and almost transparent spacer is not so interesting because of a small overall effect. This effect increases and it deserves special consideration in the case of multiple scattering of infrared radiation in a spacer. The latter circumstance explains the methodological choice made in the present study.

The schematic of the problem under consideration is shown in Fig. 1. In many cases, the 1-D problem for radiative transfer in the isothermal spacer can be rather accurately solved using the transport approximation for the scattering phase function and the traditional two-flux method [8–10]. Perhaps, the most detailed study of errors of the two-flux (or Schuster–Schwarzschild) approximation for the radiative transfer problems of this type is presented in the first chapter of the monograph [8] (see also chapter 15 of the excellent textbook [10]). It was shown that this approach enables one to determine the radiative flux very accurately because of the integral nature (over the angles) of this quantity. This statement appears to be true for an arbitrary thickness of the medium layer. The typical error in the radiative flux is less that about 5%. At the same time, it will be shown below that some corrections of the boundary condition should be made in the optically thin limit and low emissivity of the boundary surface. The mathematical problem statement for the uniform
isothermal spacer looks as the following boundary-value problem for spectral irradiance $g$ [8]

$$-D \frac{dg}{dz} + ag = 4af, \quad D = 1/(4\rho_t)$$  \hspace{1cm} (3a)

$$z = 0 \quad \frac{D g}{dz} = \frac{\epsilon t}{2}(4f - g)$$  \hspace{1cm} (3b)

$$z = d \quad \frac{D g}{dz} = \frac{\epsilon}{2}$$  \hspace{1cm} (3c)

where $z$ is the current coordinate across the spacer, $d$ is the spacer thickness, $D$ is the spectral radiation diffusion coefficient, $\rho_t = a + \sigma_t$ is the transport extinction coefficient, $a$ is the absorption coefficient, $\sigma_t$ is the transport scattering coefficient [8,9]. The spectral hemispherical emissivity $\epsilon_t$ of the spacer open surface is determined as follows:

$$\epsilon_t = -\frac{D g}{f_{1}} \bigg|_{z=d} = \frac{g}{2f_{1}}$$  \hspace{1cm} (4)

It is convenient to introduce the following dimensionless variables and parameters:

$$t_r = \beta_d z, \quad g_r = g, \quad t_\tau = \frac{\tau}{d}, \quad \sigma_t = \frac{\rho_t}{\beta_d}, \quad \xi = \frac{2\sqrt{1 - \omega}}{\omega}$$  \hspace{1cm} (5)

where $t_r$ is the transport optical thickness, $\omega$ is the transport albedo of the fibrous medium and $\xi$ is the problem eigenvalue. Using these designations, the problem (3a)–(3c) can be rewritten as follows:

$$g_r = \xi^2(g - 4)$$  \hspace{1cm} (6a)

$$g_r(0) = 2\epsilon_t(g(0) - 4) \quad g_r(t_r^0) = -2g_r(t_r^0)$$  \hspace{1cm} (6b)

where prime should be considered as a derivative with respect to $t_r$. In the case of $\xi \neq 0$, the boundary-value problem (6a)–(6b) has the following analytical solution:

$$g(r) = 4 + A \exp(\xi t_r) + B \exp(-\xi t_r)$$  \hspace{1cm} (7a)

$$A = -\frac{8\epsilon_t E_0}{(2 + \xi)(E_0^2 + \xi^2)} \quad B = \xi A$$  \hspace{1cm} (7b)

The required value of emissivity can be calculated as:

$$\epsilon_t = \frac{g(t_r^0)}{2} = 2 + \frac{AE_0 + B\epsilon_t}{2} = 2 - 2\frac{E_0^2 + \xi}{(E_0^2 + \xi^2)(2 + \xi)}$$  \hspace{1cm} (8)

This approximate relation is formally applicable at arbitrary optical thickness of the spacer. In particular, Eq. (8) gives quite correct value of $\epsilon_t$ in the limit of $t_r^0 \to \infty$ [8]:

$$\epsilon_t = 2\sqrt{1 - \omega} \left(1 + \sqrt{1 - \omega} \right)$$  \hspace{1cm} (9)

At the same time, the solution obtained is not correct in more important case of optically thin spacer. Eq. (8) gives the following formula at $t_r^0 = 0$:

$$\epsilon_t = 2\epsilon_t/(1 + \epsilon_t)$$  \hspace{1cm} (10)

instead of the correct result $\epsilon_t = \epsilon_t$. The incorrect relation (10) is explained by the simplified angular approximation (as constant values in two hemispheres) in the two-flux method. The resulting error in the vicinity of the strongly reflecting foil surface can be compensated by a formal change in the boundary condition. It is sufficient to modify the boundary condition at $t_r = 0$ as follows:

$$g(0) = 2\epsilon_t(g(0) - 4\psi)$$  \hspace{1cm} (11)

In this case, relations (7b) will be modified as follows:

$$A = -\frac{8(\epsilon_t - \psi)}{(2 + \xi)(E_0^2 + \xi^2)} \quad B = \xi A - 8\psi \quad \psi = \frac{1 - \psi}{\xi + 2\epsilon_t}$$  \hspace{1cm} (12)

and the following equation instead of (8):

$$\epsilon_t = 2\left[1 - 2\frac{(E_0^2 + \xi)(1 - \psi\epsilon_t)}{(E_0^2 + \xi^2)(2 + \xi)} - 2\psi / E_0 \right]$$  \hspace{1cm} (13)

The analytical solution (13) remains true (and gives the correct value of $\epsilon_t$ in the optical thick limit). The correction parameters $\psi$ and $\psi$ can be obtained from the following equation:

$$1/2 - \epsilon_t/4 = \frac{(1 + \xi)(1 - \psi\epsilon_t)}{(2 - \xi + \zeta(2 + \xi)) + \psi}$$  \hspace{1cm} (14)

It is obvious for Eq. (13) that the boundary condition correction is considerable only in the case of very small optical thickness of spacer and this effect is insignificant for real spacers.

3. Radiative properties of fibrous spacer

The radiative properties of a fibrous spacer can be determined on the basis of the hypothesis of independent scattering [11] and Mie theory for infinite homogeneous cylinders [8,12–14]. According to [2], the material made of fibers randomly oriented in the plane of the material layer is considered. In the heat transfer problem, the efficiency factor of absorption, $Q_a$, the transport efficiency factor of scattering, $Q_s^{\uparrow}$, and the transport efficiency factor of extinction, $Q_a^{\uparrow} + Q_s^{\uparrow}$, at arbitrary oblique illumination of fibers can be obtained by neglecting the polarization effects. In the case of fibers randomly oriented in parallel planes, it is convenient to use the efficiency factors averaged over orientations [8,14–16] as it was done in paper [2].
4. Results of calculations

Following [2], the known spectral dependences for optical constants of fused silica were used in Mie calculations. The calculations showed that the absorption band at $\lambda \approx 9 \mu m$ should be taken into account, whereas the natural uncertainty in experimental data both for the index of refraction and index of absorption of silica glass [17] is not important for the heat transfer problem under consideration. The typical averaged optical properties of glass fibers oriented in parallel planes in the most important part of the infrared spectral range are presented in Fig. 2. The choice of the wavelength range from 6 to 14 $\mu m$ in Fig. 2 is explained by the most significant contribution of this range in the integral radiative flux. It is interesting that both absorption and scattering contribute to the significant extinction in the middle-infrared.

In the case of monodisperse fibers, one can use the following relations to obtain optical properties of the fibrous spacer [8]:

$$\{ \alpha, \sigma_T, \rho_T \} = \frac{2}{\pi} \frac{f_v}{a} \left\{ Q_v, Q^T_T, Q^R_T \right\}, \quad f_v = \rho_sp_g$$

(15)

where $f_v$ is the volume fraction of fibers, $\rho_sp$ and $\rho_g$ are the densities of spacer and glass (material of fibers). Note that in the case of polydisperse fibers, one can use the so-called mean Sauter radius $a_{st}$ instead of $a$ [8,18].

The typical dependences of spectral emissivity $\varepsilon$ on fiber radius and spacer thickness at two wavelengths are shown Fig. 3. One can see in Fig. 3a that effect of fiber radius is not significant and the natural uncertainty in size distribution of glass fibers does not lead to a considerable error in value of $\varepsilon$. As one can expect, the spacer thickness is the main parameter of the problem (see Fig. 3b). In the absorption band at $\lambda = 9 \mu m$, the optically thick limit is reached at $d \approx 0.08 \mu m$ whereas the relatively thick spacer with $d=0.2 \mu m$ is not optically thick in the range of glass semi-transparency (at $\lambda = 6 \mu m$).

It is interesting that the role of the long-wave absorption band is predominant in the example problem considered (see Table 1). One can see that possible thermal contact between the spacer and one of the foils may lead to about 78% increase in the integral radiative flux even in the case of a spacer of thickness $d = 0.05 \mu m$.

In other words, local thermal contacts between a spacer and one of the adjacent foils may decrease considerably the thermal resistance of the vacuum insulation. Fortunately, this situation is expected to be temporary and may take place only in some specific parts of the space vehicle trajectory. In all cases, the effect under consideration should be taken into account in calculations of a vehicle thermal protection.

5. Comparison with experimental data

The validation of the developed theoretical model was performed by a comparison of the computational results with the laboratory experimental data. There are many technical difficulties related with measurements of both temperatures and heat fluxes at very thin layers of MLI. To overcome these difficulties a special radiative-conductive experimental technique has been developed. Just couple of elements of MLI was used as representative sample. In this case, the experimental specimens consist of two aluminum foils, which have been symmetrically heated by a thick aluminum foil installed between foils and used as an electrical heater (Fig. 4). Several highly-porous fibrous spacers (from 0 to 2) studied in paper [2] were placed between the heater and specimen, in this case some occasional contacts between spacers and foil are provided by gravitation, and it can be used as some simulation of real conditions of space flight. A schematic of the experiment at

![Fig. 2](image-url)  
Efficiency factor of absorption and transport efficiency factor of extinction for glass fibers with radius $a = 1 \mu m$ oriented randomly in parallel planes.

![Fig. 3a](image-url)  
Spectral hemispherical emissivity of aluminum foil covered by a fibrous spacer with volume fraction of fibers $f_v = 0.06$: dependences on (a) fiber radius and (b) spacer thickness.

![Fig. 3b](image-url)  
Spectral hemispherical emissivity of aluminum foil covered by a fibrous spacer with volume fraction of fibers $f_v = 0.06$: dependences on (a) fiber radius and (b) spacer thickness.

<table>
<thead>
<tr>
<th>$d$, mm</th>
<th>0 (no spacer)</th>
<th>0.05</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$, W/m²</td>
<td>38.3</td>
<td>68.2</td>
<td>69.4</td>
</tr>
</tbody>
</table>

Table 1

Variation of integral radiative flux with the spacer thickness at temperatures of aluminum foils of $T_1=500 K$ and $T_2=300 K$ (as in paper [2]).
vacuum conditions (at pressure in the range from 0.1 to 0.3 μbar) is shown in Fig. 4. Two identical heat flux sensors in the form of rectangular slabs with sizes of 80 × 80 × 3.7 mm made of opaque materials with the known thermal properties have been covered by external insulating plates from the same material and were placed symmetrically with respect to the heating element. The sensor thickness was selected in the process of pilot tests according to the temperature reaction on the back surface. The thermocouples were installed at the front (heated) and back surfaces of heat flux sensors and insulating plates. The positions of heat flux sensors and chromel-alumel thermocouples with thin wires of 0.05 mm in diameter are shown in Fig. 4. The heat flux sensors and insulated plate elements 3 and 5 have been used to estimate heat flux from aluminum foil 2. In this case, it is impossible to measure directly heat flux on the surfaces of analyzed structures especially at a significant role of thermal radiation. Therefore, an indirect measurements accompanied by a solution of an inverse heat conduction problem (IHCP) were used to estimate these heat fluxes. IHCP is an ill-posed problem in mathematical sense and the special regularizing methods are needed to solve them [1]. Additional thermocouples inside the insulating plates were used to provide the inverse problem unique solution.

The heat conduction in the heat flux sensors can be formulated as follows:

\[ C_l(T_l) \frac{dT_l}{dt} = \frac{d}{dx} \left( k_l(T_l) \frac{dT_l}{dx} \right) \]

\[ 0 \leq t \leq t_{\text{max}} \]

\[ l = 1, 2 \]

\[ T_l(X_l, 0) = T_l'(X_l), \quad X_{l-1} < x < X_l \]

\[ T_l(X_l, t) = T_l'(X_l, t) = q_l(t), \quad t_{\text{min}} < t \leq t_{\text{max}} \]

\[ k_l(T_l(X_l, t)) \frac{dT_l(X_l, t)}{dx} = k_l(T_l'(X_l, t)) \frac{dT_l'(X_l, t)}{dx} \]

\[ l = 1, 2 \]

\[ T_l^*(X_l, t) = f(t) \]

The IHCP is solved by minimization of least-square residual of calculated and measured temperatures at point of the thermocouple position [1]:

\[ J(q_l(t)) = \int_{t_{\text{min}}}^{t_{\text{max}}} \left( T_l(X_l, t) - f(t) \right)^2 dt \]

where \( T_l(X_l, t) \) is determined from a solution of the problem (16a-d).

Proceeding from the principle of iterative regularization [1], the unknown function \( q_l(t) \) can be determined through the minimization of the functional (18) by gradient methods of the first-order prior to satisfy the condition:

\[ J(q_l(t)) \leq \delta_f \]

where \( \delta_f = \int_{t_{\text{min}}}^{t_{\text{max}}} \sigma(t) dt \) is the integral error of temperature measurements.

The experimental facilities TVS-2M [3] (Fig. 5) and experimental module EM-2B (Fig. 6) were used to executed three tests of specimens with different numbers of spacers. The results of temperature measurements and the corresponded calculated temperatures are presented in Fig. 7. The results of estimating the functions \( q_l(t) \) are presented in Fig. 8. Table 2 presents the...
obtained values of the least-square and the maximum deviations of the calculated temperatures from the measured ones.

The steady-state temperature of the heater was equal to $T_1 = 673.3$ K, whereas the foil temperature appeared to be much lower ($T_2 = 454.5$ K). The latest quasi-steady-state value of estimated heat flux $q_1(t)$ can be compared with a computational prediction of heat flux through analyzed set of foils. Through the minimization of the funcented in Table 3. One can see that theoretical predictions are in good agreement with the laboratory measurements. This can be treated as a confirmation of applicability of the theoretical model developed to estimate radiative heat transfer at realistic space flight conditions, when occasional contacts of foils and spacers are possible.

6. Conclusions

A generalized spectral model for radiative transfer in vacuum thermal insulation was suggested. This model takes into account possible thermal contacts between a semi-transparent fibrous spacer and one of the neighboring aluminum foil layers.

The rigorous scattering theory was employed to calculate absorption and scattering properties of single glass fibers and to obtain the volumetric optical properties of the layered spacer made of thin glass fibers. The continuous approach and two-flux model with a slightly modified boundary condition were used to derive an analytical solution for the radiative heat flux between the foils.

The computational results appeared to be in good agreement with the laboratory thermal measurements at vacuum conditions. This validation enables us to recommend the suggested analytical model for engineering estimates of thermal effects of possible temporary contacts between the spacer and adjacent aluminum foils on the quality of a multi-layer thermal insulation at space flight conditions.
Conflict of interest

None declared.

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