Directional reflectance of optically dense planetary atmosphere illuminated by solar light: An approximate solution and its verification

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A B S T R A C T
A new combined approach to calculate the spectral directional reflectance of optically thick cloudy atmosphere illuminated by solar light is suggested. First of all, the approximate method based on the idea of previously developed two-step procedure and a set of 1-D solutions is employed to calculate the local bi-directional reflectance of the cloud layer. After that, the Monte Carlo ray-tracing procedure is used to determine the reflectance of the planet as a whole in the case of a remote sensing from a space vehicle at large distances from the planet. This combined approach appeared to be applicable for planets with opaque cloudy atmosphere. A comparison with the reference Monte Carlo simulation of the complete problem in a wide range of volumetric optical parameters typical of Venus atmosphere in the visible and infrared spectral ranges confirmed very good accuracy of suggested approach. It is recommended to consider the solution obtained as an alternative multi-wavelength method in navigation systems of space vehicles.

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1. Introduction

The present paper is concerned with scattering of solar radiation by optically dense planetary atmosphere observed from a large distance from the planet when the visible angular diameter of the planet is small and one can neglect the difference between the directions of light beams coming from various parts of the atmosphere. For simplicity, the optical properties of the external part of a spherical cloud layer are assumed to be uniform. This enables us to focus on the method of calculations, but the method developed can be generalized to the case of a nonuniform atmosphere.

The atmosphere of Venus seems to be appropriate for the use of the method suggested. Therefore, we considered the simplified model of the Venus upper atmosphere as the case problem. It should be recalled that the orbital flights around the Venus are often used in space missions [1–3]. Therefore, an additional orientation system based on the direction and value of solar radiation flux reflected by Venus may be interesting for the potential use in navigation systems of space vehicles. Note that Venus is much brighter than any other planet in the Earth’s sky. It is the third-brightness object in the sky, after the sun and moon. The orientation in space using the light reflected by Venus is a specific engineering problem which is not considered in the paper. The details of the reflection by different parts of the cloudy atmosphere are not important to solve the problem at large distances from the planet, and the main parameters such as the observation angle and the distance from the planet can be estimated on the basis of a relatively simple model of the upper cloudy atmosphere.

The objective of the present work is two-fold: (1) to develop a simplified semi-analytical method for computational prediction of the spectral directional reflection of solar radiation by Venus observed from a large distance and (2) to examine the accuracy of this method in a wide spectral range using the reference Monte Carlo calculations.

The cloudy atmosphere of Venus has been studied by several research groups over the world during many years [4–19]. It appears that the clouds containing mainly droplets of sulfuric acid are not uniform and contain two or even three modes of droplets with radii of 0.1–0.2 μm, 0.8–1 μm and about 3.5 μm [17,18]. In addition, the particle size are 1.5–2 times smaller in the polar regions [16,17]. It would be a separate task to overview the published paper and detailed data for Venus clouds reported there, but it is beyond the scope of the present study focused on the simplified approach which is expected to be useful for the space vehicle orientation based on observation of both the Sun light and its spectral reflection from the Venus. At the same time, a brief analysis of the
optical properties of sulfuric acid droplets typical of Venus clouds is given below to estimate the range of the problem parameters.

2. Spectral optical properties of sulfuric acid droplets

The data reported by Palmer and Williams [20] for spectral optical properties of H$_2$SO$_4$ were used in the calculations. The spectral variation of both index of refraction, $n$, and index of absorption, $k$, in the near-infrared range is shown in Fig. 1. The sulfuric acid is almost transparent in the visible range, whereas the index of absorption increases sharply with the wavelength in the near-infrared (Fig. 1b).

The classical Mie theory for absorption and scattering of light by a homogeneous spherical particle [21–23] is used in the calculations for single droplets. Possible electromagnetic interaction of closely positioned droplets is ignored. The latter assumption is known as the hypothesis of independent scattering [24–26]. According to this widely used approach, each particle is assumed to absorb and scatter the radiation in exactly the same manner as if other particles did not exist. In addition, there is no systematic phase relation between partial waves scattered by individual particles, so that the intensities of the partial waves can be added without regard to phase. In other words, each particle is in the far-field zones of all other particles, and scattering by individual particles is incoherent. The computer code described in [27] was used in all the calculations.

In the present paper, the model problem for uniform optically thick clouds characterized by multiple scattering is considered. As a result, there is no need in absolute values of absorption and scattering parameters and the only physical parameter affects the cloud reflectance. This is the so-called transport albedo of scattering. The word “transport” means that we use the transport approximation of the scattering phase function which is presented as a sum of an isotropic part and a peak in the forward direction [28]. In this approach, the so-called asymmetry factor of scattering, $\mu$, is sufficient for reliable radiative transfer calculations [27]. The multiple scattering in a semitransparent optically thick medium is really favorable condition for the use of transport approximation because the details of angular distribution of light intensity in single scattering have a negligible effect on the total cloud reflectance [27–29]. The spectral value of transport albedo of single droplets, $\omega_{tr}$, shown in Fig. 2 is defined as follows:

$$\omega_{tr} = Q^i/Q_s, \quad Q_s = Q_a + Q_{tr}$$

$$Q_{tr} = Q_s(1 - \mu)$$

(1)

where $Q_a$ and $Q_s$ are the ordinary efficiency factors of absorption and scattering [21,22].

The calculations showed that transport albedo of sulfuric acid droplets is not only extremely large in the visible spectral range, where one can use the value of $\omega_{tr} = 1$ for all the droplets, but $\omega_{tr}$ is also rather large in the near-infrared range, excluding the case of very small sub-micron droplets, which optical properties are described by the Rayleigh theory. The resulting large transport albedo of cloud droplets explains the observed high reflectance of solar radiation by the Venus atmosphere.

3. Simplified model for radiative transfer

The known observations confirmed that the reflectance of optically thick cloudy atmosphere like the upper atmosphere of Venus is rather high. It means that scattering of solar light in clouds is much greater than the light absorption and the multiple scattering takes place in the cloud. In the case of Venus-like atmosphere, which is characterized by a vigorous circulation, the superrotation, and hurricane-force winds [30,31], one can use another important simplification. It is assumed in the present paper that the global cloud layer can be considered as a spherical one and an average angular variation of the atmosphere optical properties is relatively small in arbitrary direction. Strictly speaking, the atmospheric circulation of Venus is rather complex, and includes a big double-vortex structure at the Venus South pole [32], but the overall reflectance at large distances from the planet is expected to be weakly sensitive to these effects. So, it is assumed that good estimates of reflectance of Venus atmosphere can be obtained by accounting for only the radial variation of optical properties in a relatively thin spherically symmetric layer of clouds. Moreover, the clouds are optically dense, and one can consider a set of thin discrete cells formed by a grid of latitude and longitude lines. These thin computational cells are almost flat. Therefore, the 1-D approach for radiative transfer in a single cell at oblique incidence of solar radiation can be considered by neglecting the solar light propagation along the cloud surface. Note that the use of a set of 1-D solutions instead of direct solving the original multi-dimensional problem is not a novelty, and this approach appeared to be sufficiently good in quite different applications [33–35].

The distance from the Sun to the terrestrial planets is much greater than the Sun diameter. Therefore, the sunlight is assumed to be collimated. Of course, the real problem is a spectral one, but mathematical formulation of the spectral radiative transfer prob-
lem is universal. In the case of collimated incidence, the radiation field is three-dimensional. However, this problem can be replaced by an axisymmetric problem of the oblique irradiation along the cone surface with the same angle of incidence. It is sufficient to rotate the original schematic of the 3-D problem around the normal to the plane-parallel layer of a medium to make the latter statement obvious. The axial symmetry enables one to integrate the original radiative transfer equation (RTE) over the azimuth angle. The resulting RTE and the boundary conditions for the scattering cloud layer can be written as follows [27,36,37] (subscript \( \lambda \) is hereafter omitted for brevity):

\[
\frac{\partial \tilde{I}}{\partial \tau_{\text{tr}}} + \tilde{I} = \frac{\omega_{\text{tr}}}{2} \int_{-1}^{1} \tilde{I}(\tau_{\text{tr}}, \mu) d\mu \quad \mu = \cos \theta \quad \tau_{\text{tr}} = \int_{0}^{z} \beta_{\text{tr}}(z) dz
\]

(2a)

\[
\tilde{I}(0, \mu) = \delta(\mu - \mu_i) \quad \tilde{I}(\infty, -\mu) = 0 \quad \mu, \mu_i > 0
\]

(2b)

where the coordinate \( z \) is measured from the outer surface of the cloud layer along the local external normal \( (\theta = 0) \), \( \mu_i = \cos \theta_i \) is

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**Fig. 1.** Spectral optical constants of sulfuric acid in the near-infrared range [18]: a – index of refraction, b – index of absorption.

**Fig. 2.** Transport albedo of sulfuric acid droplets in the near infrared: a – spectral dependences, b – effect of droplet radius.
the direction of incident solar light, $\sigma_{Ir} = \sigma \cdot (1 - \bar{\mu})$ is the transport scattering coefficient ($\sigma$ is the ordinary scattering coefficient), $\beta_{Ir} = \sigma + \sigma_{Ir}$ is the spectral transport extinction coefficient, $\alpha$ is the spectral absorption coefficient, $\tau_{Ir}$ is the current transport optical thickness, $\omega_{Ir}(\tau_{Ir}) = \alpha_{Ir}/\beta_{Ir}$ is the transport albedo of the medium, $\bar{I}(\tau_{Ir}, \mu) = I(\tau_{Ir}, \mu)/q_{0\mu}$ is the normalized (per unit incident radiative flux) spectral radiation intensity. For simplicity, Eq. (2a) is written for the case of an isotropic medium when the coefficients of RTE do not depend on direction.

Following the usual technique [38], consider the radiation intensity $\bar{I}$ as a sum of the diffuse component $\bar{f}$ and the term, which corresponds to the reflected directional solar radiation:

$$\bar{I}(\tau_{Ir}, \mu) = \bar{f}(\tau_{Ir}, \mu) + E_{Ir}\delta(\mu - \mu_{i})$$

$$E_{Ir} = \exp(-\tau_{Ir}/\mu_{i})$$

$$\tau_{Ir,i} = \tau_{Ir}/\mu_{i}$$

(3)

Note that the specific case of oblique collimated incidence of solar radiation on a spray of water droplets has been already considered in paper [39].

The mathematical problem statement for the diffuse component of radiation intensity is as follows:

$$\frac{\partial \bar{f}}{\partial \tau_{Ir}} + \bar{f} = \frac{\omega_{Ir}}{2}(G + E_{Ir,i})$$

$$G = \int_{0}^{1} \bar{f} d\mu$$

(4a)

$$\bar{f}(0, \mu) = \bar{f}(\infty, -\mu) = 0$$

$$\mu > 0$$

(4b)

The directional-hemispherical reflectance of solar radiation can be expressed through the diffuse component of the radiation intensity:

$$R_{\mu_i} = \int_{0}^{1} \bar{f}(0, -\mu) \mu d\mu$$

(5)

It is important that the source term on the right of Eq. (4a) does not depend on angular variable $\mu$. According to Dombrovsky et al. [40,41], it enables further simplification of the problem with the use of two-flux approximation which can be treated as a reduced modified two-flux model for a refracting medium [27]. The following presentation of the spectral radiation intensity is considered:

$$\bar{f}(\tau_{Ir}, \mu) = \begin{cases} \bar{f}^{+}(\tau_{Ir}), & -1 < \mu < 0 \\ \bar{f}^{-}(\tau_{Ir}), & 0 < \mu < 1 \end{cases}$$

(6)

Integrating Eq. (4a) separately over the intervals $-1 < \mu < 0$ and $0 < \mu < 1$, after simple transformations, one can obtain the following boundary-value problem for the irradiation function $G(\tau_{Ir}) = \tilde{f}^{+}(\tau_{Ir}) + \tilde{f}^{-}(\tau_{Ir})$:

$$-G'' + \xi^2 G = 4\omega_{Ir}E_{Ir,i}$$

$$\xi = 2\sqrt{1 - \omega_{Ir}}$$

$$G'(0) = 2G(0)$$

$$G'(\infty) = 0$$

(7a)

$$G'(0) = 2G(0)$$

$$G'(\infty) = 0$$

(7b)

where $\xi$ is the eigenvalue of a similar problem for the homogeneous equation $-G'' + \xi^2 G = 0$. Strictly speaking, one should use a large but finite value of the argument in the second of the boundary condition (7b), but the solution will give the same result in the limit of an optically thick cloud. Note that Eq. (5) leads to the following expression for the directional-hemispherical reflectance:

$$R_{\mu_i} = \bar{f}^{-}(0)/(2\mu_i) = G(0)/(2\mu_i)$$

(8)

The problem (7a) and (7b) can be solved numerically at arbitrary dependence of $\omega_{Ir}(\tau_{Ir})$. At the same time, the analytical solution at a constant value of $\omega_{Ir}$ has two obvious advantages. First of all, it is convenient for verification of the approximate solution. It is important also that the model of uniform optical properties of an external layer of the Venus clouds can be used to retrieve these conventional/equivalent spectral properties using the optical remote sensing. In the case of $\xi = 1/\mu_i$, the analytical solution to the boundary-value problem (7a) and (7b) can be written as follows:

$$G(\tau_{Ir}, \mu_i) = \frac{4\omega_{Ir}}{\xi^2 - 1/\mu_i^2} \left( E_{Ir,i} - \frac{2 + 1/\mu_i}{2 + \xi} \exp\left(-\xi \tau_{Ir}\right) \right)$$

(9)

where $\mu_i$ should be considered as a parameter. Eq. (9) gives the following formula for the value of irradiation at the cloud boundary:

$$G(0, \mu_i) = \frac{4\omega_{Ir}}{2 + \xi}(1 + \mu_i \xi)$$

(10)

The limiting values of $G(0, \mu_i) = 0$ at $\omega_{Ir} = 0$ and $G(0, \mu_i) = 2\mu_i$ at $\omega_{Ir} = 1 (\xi = 0)$ are physically correct. The following expression for the directional-hemispherical reflectance which is also correct at arbitrary incidence can be obtained:

$$R_{\mu_i} = \frac{\omega_{Ir}}{(1 + \xi/2)(1 + \mu_i \xi)}$$

(11)

At normal incidence ($\mu_i = 1$), Eq. (11) gives the known expression for the normal-hemispherical reflectance derived in paper [41] (a general expression for the refractive host medium was derived in [41]):

$$R_{\mu_i} = \frac{\omega_{Ir}}{(1 + \xi/2)(1 + \xi)}$$

(12)

Note that only the hemispherical reflectance can be determined with the use of the two-flux model because of specific of angular approximation (6), but this reflectance value is insufficient for the second step of the two-step model to obtain the local bi-directional reflectance. As to the derived approximate solution for the irradiation function, it should be used to determine the RTE source function. The latter enables one to employ an analytical ray-tracing procedure which is similar to that for non-scattering media.

The simplest example illustrating a sufficiently high accuracy of the above described two-step procedure has been presented in paper [42] where the normal emittance of the absorbing, scattering, and refracting medium was calculated. In the case of a non-refracting medium, the error of this method appeared to be negligible, and practically the total coincidence of approximate analytical calculations with the reference solutions based on both the high-order discrete ordinate method and the Monte Carlo method over the whole range of the medium transport albedo and optical thickness was obtained. According to Dombrovskiy and Lipinski [43] and Dombrovsky et al. [44], the use of transport approximation and simple differential approach to determine the source function in the RTE is also a promising way to simplify the complete Monte Carlo calculations even in some multi-dimensional problems. Therefore, a similar approach is used in the present paper.

The RTE for the radiation intensity $\bar{f}$ scattered by a cloud cell to the space in the direction $\mu_i > 0$ can be written as follows:

$$-\mu_i \frac{\partial \bar{f}_{\mu_i}}{\partial \tau_{Ir}} + \bar{f}_{\mu_i} = \frac{\omega_{Ir}}{2}(G + E_{Ir,i}) \bar{f} (\infty, \mu_i) = 0$$

(13)

where the irradiation function is given by the approximate analytical solution (9) and $\mu_i$ is considered as a parameter of function $\bar{f}(\tau_{Ir}, \mu_i)$. The value of this function at $\tau_{Ir} = 0$ can be obtained by the integration along the ray:

$$\bar{f}(0, \mu_i) = \frac{\omega_{Ir}}{2\mu_i} \int_{0}^{\infty} [G(\tau_{Ir}, \mu_i) + E_{Ir,i}] d\tau_{Ir}$$

(14)

It is convenient also to use the normalized value of $\bar{f}(0) = \bar{f}(0, \mu_i)/\mu_i$ which takes into account the effect of the incidence angle on the incident radiative flux. Having substituted Eqs. (9)–(14), one can find the following analytical expression for
the dimensionless radiation intensity coming out from the cloud cell:

\[ R_j (0) = \omega_{tr} \frac{\mu_i + 1/2}{2 + \xi \mu_j} \left( \frac{2 \mu_i - 1}{\mu_i + \mu_j} \right) \left( \frac{1 + \xi \mu_j}{1 + \xi \mu_i} \right) \]  

In the realistic case of a highly scattering medium, when \( 1 - \omega_{tr} < \xi < 1 \), Eq. (15) is simplified as follows:

\[ R_j (0) = (2 \mu_i + 1) \left( 1 - \mu_i \frac{\mu_i - 1/2}{\mu_i + \mu_j} \right) \]  

One can estimate an accuracy of the angular dependence on \( \mu_j \) given by Eq. (16) at the simplest case of normal incidence (\( \mu_i = 1 \)):

\[ R_{j, h} = \int \int \frac{3 \mu_j + 2 \mu_i}{2 + \mu_j} \]  

This estimate is based on comparison of the calculated value of normal-hemispherical reflectance with the correct value of \( R_{j, h} = 1 \). The following result can be obtained with the use of Eq. (17):

\[ R_{j, h} = \int_0^1 \int_0^1 \omega_{tr} \mu_i d\mu_j = 1.5 \ln 2 \approx 1.04 \]  

The above estimate indicates that the integral error of the two-step analytical solution of about 4%.

It should be mentioned that we consider the axisymmetric problem with the incident radiation propagating along the cone surface. Therefore, the bi-directional reflectance introduced in paper [45], is calculated as follows:

\[ R_{j, j} = \int_0^1 \int_0^1 (0) (2\pi) \]  

It is interesting that the above derived analytical solution gives constant value of \( R_{j, j} = 1/\pi \) (independent of \( \mu_j \)) for highly scattering media at \( \mu_i = 0.5 \). This result is convenient for the method verification.

Some results of \( R_{j, j} \) calculations are presented in Figs. 3 and 4. One can see that transport albedo of cloud of highly-scattering droplets is the strongest parameter of the problem and even small absorption leads to a significant decrease in the reflectance. A comparison of analytical solution for bi-directional reflectance obtained using the two-step analytical procedure and Monte Carlo benchmark calculations indicates that the approximate analytical solution is rather accurate to determine the bi-directional reflectance at normal incidence, whereas the relative error may reach from \(-4\%\) to about \(+10\%\) at oblique incidence. The latter seems to be acceptable for practical calculation of bi-directional reflectance in the problem under consideration because a contribution of this error to the calculated total reflection from Venus is expected to be insignificant. A relatively large error of the combined method is similar to that obtained in paper [43] for the so-called searchlight effect, which appeared to be the most difficult case for the simplified calculations.

Having in mind that \( P_i \) approximation of the method of spherical harmonics gives better results than the two-flux method at large optical distances from the illuminated surface [27], the results obtained using \( P_i \) at the first step of the calculations, are also shown in Fig. 4. One can see that a choice of the simple differential approximation has an insignificant effect on the calculated bi-directional reflectance. A contribution of the bright part of the scattering Venus clouds is expected to be the most important. Therefore, we will use the two-flux method in subsequent calculations.

Most likely, the errors of the approximate analytical solution for the bi-directional reflectance at some combinations of the incidence and reflection angels may lead to insufficiently accurate results even for the overall reflectance at small distances from the planet. However, it will be shown below that the observations of the planet reflectance from large distances are weakly sensitive to these errors.

4. Observation from the large distance

It the case of a remote sensing, the general problem is radically simplified because it is sufficient to consider the sunlight reflection in the only direction to the sensor of the space vehicle. The axial
symmetry of the problem makes possible to consider only two parameters of the problem: the distance between the space vehicle and the Venus, $R_{\text{obs}} > > R_{\text{v}}$, and the observation angle, $\gamma_{\text{obs}}$. Moreover, the observed radiative flux is inversely proportional to the $R_{\text{obs}}^2$. Therefore, it is sufficient to consider a relative value of reflectance as a function of the observation angle. Most likely, this approach can be used when $R_{\text{obs}} > 10R_{\text{v}}$.

Let us define the angular coordinates of the above-introduced cells on the spherical cloud surface. It is convenient to consider two orthogonal planes of solar illumination symmetry, $\psi = 0$ and $\psi = \pi / 2$ shown in Fig. 5, so that the sun is at its zenith at the intersection of these planes. The other angular coordinate, $\varphi$, is measured from the zenith point in the plane of $\psi = \pi / 2$. The axial symmetry of the problem enables us to assume that the observation direction coincides with the plane of $\psi = 0$ and to introduce the observation angle, $0 < \gamma_{\text{obs}} < \pi$, between the illumination and observation directions. At the same time, the position of a cloud cell on spherical surface of the cloud layer is determined by two orthogonal angular variables: $\varphi$ and $\psi$.

Only the hemisphere $0 < \psi < \pi / 2$ $(0 < \psi < 2\pi)$ is illuminated by the Sun. Therefore, the sensor of a remote space vehicle can receive the scattered light from a part of this hemisphere only. It the case of $\gamma_{\text{obs}} = 0$, the illuminated hemisphere is invisible, whereas the opposite case of $\gamma_{\text{obs}} = \pi$ corresponds to the maximum observed scattering. The observation angle can be determined using the angle $\pi - \gamma_{\text{obs}}$, which is equal to the angle between the incidence and observation directions (see Fig. 5), and a choice of local or global coordinate system makes no difference. In both cases, we can use the following scalar product:

$$\cos\,\gamma_{\text{obs}} = -(\vec{\mu}_i, \vec{\mu}_j)$$

where $\vec{\mu}_i$ and $\vec{\mu}_j$ are the unit vectors in the incidence and scattering directions, respectively. In the local coordinates, the following relations take place:

$$\vec{\mu}_i = \left(\cos\psi \sin\varphi \cos\varphi, \sin\psi \sin\varphi, \cos\psi \right)$$

$$\vec{\mu}_j = \left(\cos\psi \sin\varphi \cos\varphi, \sin\psi \sin\varphi, \cos\psi \right)$$

The retrieval of the unknown local scattering angle, which satisfies the condition (20), appears to be a very complex problem. Therefore, the Monte Carlo procedure was employed for the ray-tracing not only in complete reference calculations but also in calculations based on analytical solution for the local bi-directional reflectance of the clouds.

The forward collision-based Monte Carlo method is employed in the present paper. The details of this method can be found in textbooks [36,37], in review paper [46], and in previous publications by the authors [33,42,47]. The key steps of the method applied to the simulation of radiative transfer in optically thick cloudy atmospheres are described below. Note that Monte Carlo method has been used in other studies for solving the radiative transfer problem in cloudy atmospheres [48–50]. The field of radiation inten-
sity, and consequently the directional spectral reflectance, are determined statistically.

In the forward Monte Carlo algorithm of simulation of ray propagation in an absorbing and scattering medium, a large number of stochastic incident rays $N_{\text{rays}}$ of direction cosine $\mu_i$ with respect to the outward normal at the irradiated boundary of the computational region is followed throughout the medium until either they leave the medium at the same boundary ($\tau_n = 0$) or are absorbed within the medium. In the problem under consideration, the cloudy layer is optically thick and no ray bundle exit the medium at the shadow boundary of the computational region at $\tau_n = \tau_n^0$, i.e. $z = L$, where $L$ is the thickness of the layer. The path length of a ray bundle before extinction inside the medium is determined as follows:

$$s = -\frac{1}{\beta} \ln R_s, \quad \beta = \frac{\tau_0[1 - \tilde{\beta}(1 - \omega_0)]}{L(1 - \tilde{\beta})}$$  \hspace{1cm} (22)

At the attenuation location, the type of extinction is tested. In the case of $R_s < \omega_0$ where $\omega_0 = \omega_0/(1 - \tilde{\beta}(1 - \omega_0))$, the ray is absorbed and its history is terminated. Otherwise, the ray is scattered and the scattering direction is determined by the values obtained from the equations:

$$R_\mu = \int_{-1}^{1} \Phi (\mu') d\mu' / \int_{-1}^{1} \Phi (\mu) d\mu \quad \varphi = 2\pi R_\varphi$$ \hspace{1cm} (23)

where $\Phi (\mu)$ is the phase function of sulfate aerosol droplets. The values of $R_s, R_o, R_p$ and $R_\mu$ are the different random numbers taken from a uniform distribution over the interval of $(0,1)$.

After the scattering, the ray continues its path with the new direction. The above extinction and scattering tests are repeated until the ray is absorbed in the medium or leaves the cloud layer. In the latter case, a new ray is considered. A ray leaving the medium at the boundary by $\tau_n = 0$ contributes to the reflectance. The direction vector of the ray leaving the cloud at $\tau_n = 0$ and local angles $\mu_i$ and $\psi_j$ with respect to the external normal is calculated. The observation angle $\gamma_{\text{obs}}$ can be determined from known values of local angles related to incident ray ($\mu_i$ and $\psi_j$) and ray leaving the medium ($\mu_i$ and $\psi_j$). Each time a ray leaves the medium, the two following parameters are updated: $N(\mu_i)$ and $N(\gamma_{\text{obs}})$. The first parameter refers to the number of rays leaving the medium in unit solid angle $\Delta \Omega$ around the direction $\mu_i$. The second parameter corresponds to the similar values of $\Delta \Omega_{\text{obs}}$ and $\gamma_{\text{obs}}$. After all $N_{\text{rays}}$ have been traced, the bi-directional reflectance and the observed reflectance are calculated as:

$$R_\mu = \int_{-1}^{1} \Phi (\mu') d\mu' / \int_{-1}^{1} \Phi (\mu) d\mu \quad R_{\text{obs}} = \frac{N(\gamma_{\text{obs}})}{N_{\text{rays}} \Delta \Omega_{\text{obs}}}$$ \hspace{1cm} (24)

An analysis of the influence of the ray number $N_{\text{rays}}$ shows that $N = 10^7$ gives the numerical error less than 1% with respect to the results for a very large number of rays ($10^8$ here).

The suggested analytical solution for bi-directional reflectance of Venus clouds is a basis of an alternative two-step computational method. The formulas (11) and (15) for the directional-hemispherical and bi-directional reflectances are also used in these calculations. Eq. (11) is used to test if the ray is reflected by the cloud at irradiated boundary or absorbed in the medium, whereas Eq. (15) is used in the Monte Carlo procedure as a probability function to determine the local reflection angle from the known incidence angle. The details of subsequent Monte Carlo calculations are omitted here for brevity. A comparison of the reference Monte Carlo simulation and the two-step method is presented in Fig. 6. There is very good agreement of the computational results obtained using the suggested combined method and the reference solution in a wide range of the problem parameters including those typical for the infrared observations. At the same time, the calculations using the semi-analytical procedure take much less CPU time than that in the reference numerical solution. The difference in computational time is from about four times to the order of magnitude. Specifically, with a desktop computer Intel(R) Xeon(R) CPU E5-2637 v3 @ 3.5 GHz and 128 Go RAM, the computational time associated with the two-step method is 15 s and 30 s for albedo values of 0.95 and 0.999. The corresponding computation times of Monte Carlo simulation are 1 min and 6 min, respectively.

The results obtained can be treated as a possibility of a faster space navigation. Most likely, the multi-wavelength measurements could be recommended for the reliable navigation system. The measurements of this type can be also used for other planets with an optically dense cloudy atmosphere to estimate spectral behavior of transport albedo of an upper layer of the clouds.

It is important that a strong effect of the transport scattering albedo of the cloud medium on the measured spectral reflectance of solar light by a cloudy planet atmosphere makes realistic an approximate retrieval of optical properties and chemical composition of cloud particles of upper atmosphere. Of course, this is a special task and it is beyond the scope of the present paper.

5. Conclusions

A combined analytical and numerical approach to calculate the spectral directional reflectance of optically thick cloudy atmosphere illuminated by solar light is suggested. The first analytical step of solution is based on both the transport approximation and a set of 1-D two-flux solutions. This makes possible to decrease the computational time from about four times up to the order of magnitude as compared with the complete numerical solution. The calculated reflectance observed from a large distance of a Venus-like planet is compared with the reference Monte Carlo solution. This comparison confirmed very good accuracy of the suggested approach both in the visible and infrared spectral ranges. Therefore, the multi-wavelength measurements could be recommended for more reliable space navigation system. The measurements of this type can be used for various planets with an optically dense cloudy atmosphere to estimate spectral behavior of transport albedo of an upper layer of the clouds. In this case, one could retrieve an additional preliminary information on optical properties and chemical composition of cloud particles.

Conflict of interest

None declared.
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